Chemical responses to hydraulic fracturing and wolframite precipitation in the vein-type tungsten deposits of southern China

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Keywords: Hydraulic fracturing, Tungsten deposits, Wolframite precipitation, CO₂ loss, Thermodynamic model, Numerical model

Abstract

Wolframite is the main ore mineral in the vein-type tungsten deposits of southern China. Much progress has been made on the characteristics of the mineralizing fluids, but the mechanisms of wolframite precipitation remain poorly understood. Hydraulic fracturing driven by high-pressure fluids is a common mechanical process during magmatic-hydrothermal transition, but it is uncertain whether and how this mechanical process may affect chemical equilibrium and cause wolframite precipitation. This paper examines how a hydraulic fracturing process affects solubility of tungsten in CO₂-saturated NaCl solutions using a hydro-mechanical numerical model coupled with a multi-component thermodynamic model. The thermodynamic model presented here is in the system of Fe-W-CI-Na-C-O-H. The modeling results indicate that fluid pressure exerts a significant influence on chemical equilibrium where CO₂ solubility in NaCl solutions decreases with decreasing fluid pressure and pH increases with decreasing fluid pressure. An increase in pH reduces the concentrations of the dominant iron-bearing species (FeCl₂) and the dominant tungsten-bearing species (HWO₄⁻) in fluids. Tungsten solubility in fluids reaches tens of ppm. Over ten fluctuations of fluid pressure are identified in the numerical experiments of hydraulic fracturing. These pressure fluctuations cause a decrease in solubility of tungsten by over 30% of the maximum solubility. Repeated drops of fluid pressure during hydraulic fracturing processes cause CO₂ loss and could be efficient processes for precipitating wolframite from mineralizing fluids. These findings may also offer an insight into the precipitation mechanisms of other metals from CO₂-bearing hydrothermal fluids.

1. Introduction

Magmatic-hydrothermal fluids are efficient agents of metal transport and form ore deposits with economically exploitable tungsten, tin, copper, and gold (Černý et al., 2005). How these metals are precipitated from magmatic-hydrothermal fluids is critical for understanding the ore forming processes (e.g. Audétat et al., 1998; Heinrich, 1990; Heinrich et al., 2005; Seward, 1997; Williams-Jones and Migdisov, 2013). Mixing of magmatic fluids and meteoric fluids is an efficient way to precipitate ore minerals (e.g. Audétat et al., 1998; Fekete et al., 2016; Heinrich, 2007). However, release of hydrothermal fluids from a volatile-rich magma suppresses free convection and prevents meteoric fluids from mixing with magmatic fluids close to the magma (e.g. Gerdes et al., 1998; Hanson, 1995; Weis et al., 2012). In this case, other possible mechanisms of precipitating ore minerals include simple cooling, phase separation, fluid-rock interaction, and hydraulic fracturing (e.g. Gibert et al., 1992; Korges et al., 2017; Lecumberri-Sanchez et al., 2017; Ni et al., 2015; Polya, 1990). The efficiency of metal precipitation reached by simple cooling is severely limited by the low rate of heat transfer to surrounding rock (Barton and Toulmin, 1961; Heinrich et al., 2005). Phase separation is generally triggered in shallower hydrothermal systems where fluid pressure decreases to the two-phase boundary (e.g. Korges et al., 2017). Fluid-rock interaction requires specific chemical disequilibrium between hydrothermal fluids and rocks (e.g. Lecumberri-Sanchez et al., 2017). Hydraulic fracturing may also cause metal precipitation from CO₂-bearing hydrothermal fluids (e.g. Polya, 1990), but it remains poorly understood how this fluid-driven mechanical process affects chemical equilibrium.

Wolframite is the main ore mineral in the vein-type tungsten deposits of southern China, which is a world-class tungsten province (Mao...
et al., 2013). How wolframite in these deposits is precipitated from hydrothermal fluids is highly disputed (e.g. Ni et al., 2015; Wei et al., 2012; Xiong et al., 2017). Wei et al. (2012) proposed that mixing of magmatic fluids and meteoric fluids is the dominant mechanism for precipitating the wolframite in the Xihuashan tungsten deposit. However, many other hydrogen and oxygen isotopic measurements suggest that the mineralizing fluids were not mixed with meteoric fluids at the main mineralization stage (e.g. Gong et al., 2015; Liu et al., 2011; Mu et al., 1981; Wang et al., 2007; Zhu et al., 2015). Ni et al. (2015) argued that simple cooling caused precipitation of wolframite, but heat transfer from hydrothermal fluids to wallrock is too slow to precipitate wolframite efficiently (cf. Barton and Toulmin, 1961). The third mechanism is CO2 escaping and pH increase (Wang et al., 2012; Wang et al. 2013b; Xiong et al., 2017). Recent infrared micro-thermometric studies found CO2 from fluid inclusions in wolframite (e.g. Chen et al., 2018; Li et al., 2018). Previous structural and geochemical studies suggest high-pressure fluids are required for triggering fracture initiation and propagation and the fluctuations of fluid pressure are recorded by fluid inclusions (e.g. He and Xi, 1988; Wang et al., 2008; Xi et al., 2008; Yu, 2004). Thus, it is necessary to examine whether and how CO2 loss accompanying a decrease in fluid pressure causes precipitation of wolframite.

The thermodynamic model proposed by Wood and Samson (2000) provides a fundamental understanding of the solubilities of ferberite and scheelite in CO2-free NaCl solutions. In this study, a thermodynamic model including CO2 and its reactions in NaCl aqueous solutions was developed. An important difference from the results in Wood and Samson (2000) is that tungsten solubility in the models presented here decreases with fluid pressure. It was found that fluid pressure fluctuations during a hydraulic fracturing process could precipitate a great part of tungsten dissolved in hydrothermal fluids.

2. Geological background

The South China tectonic block (SCB) consists of the Cathaysian Block and the Yangtze Craton (Fig. 1). The tectonic setting in the SCB was largely under extension during the Late Mesozoic (J2-K3) (Zhou et al., 2006), during which time NE-trending extensional lithospheric belts and deep faults controlled large-scale magmatic activities and related tungsten mineralization (Huang and Jiang, 2014; Mao et al., 2013; Zhao et al., 2017b). The tungsten mineralization in this region is dominated by quartz-vein, skarn, and greisen types (Zhao et al., 2017b). The vein-type tungsten deposits are the subject of this study.

The tungsten-bearing quartz veins are hosted by the Neoproterozoic to Jurassic strata and granitic rocks and extend a depth of approximately 1000 m (see Fig. 2) (Gu, 1984; Zhu et al., 1981). Wolframite is the main ore mineral in the veins (Fig. 3), and the gangue minerals have quartz, feldspar, muscovite, calcite, and tourmaline (Chen et al., 1989). It is interpreted that the mineralizing fluids have a magmatic origin at the main mineralization stage, and then were mixed with meteoric waters at late stages (Zhu et al., 2014). A few lines of evidence suggest that these deposits may have a genetic relationship to highly fractionated granitoids at deeper levels (cf. Fang et al., 2015; Legros et al., 2018; Wang et al., 2017; Wu et al., 2017; Zhao et al., 2017b).

The quartz veins in most tungsten deposits are sub-vertical and have a preferred orientation of approximately east-west striking (Fig. 2) (Zhu et al., 1981). The sheeted veins often show a vertically morphological zonation that the thin veins at shallower levels become thicker downwards (Liu et al., 2014; Liu and Ma, 1993). The mineralizing fluids are NaCl-H2O ± CO2 systems. The fluid inclusions trapped by ore and gangue minerals have homogenization temperatures from 160 °C to 390 °C and salinities from 1 to 10 wt% NaCl equivalent. The mineralization pressures recorded by fluid inclusions have a range from 20 to 160 MPa with maximum range of 75–160 MPa (Gong et al., 2015; Liu et al., 2011; Mu et al., 1981; Ni et al., 2015; Wang et al., 2007; Wang et al. 2013a; Wei et al., 2012; Xi et al., 2008; Xiong et al., 2017; Zhou et al., 2017; Zhu et al., 2015). CO2-bearing fluid inclusions are recorded in wolframite, topaz, and quartz (e.g. Chen et al., 2018; Li et al., 2018; Wang et al., 2012; Xi et al., 2008; Xiong et al., 2017; Zhou et al., 2017). Also, fluid inclusions in quartz have high concentrations of W, Fe, Mn, and other ore elements (Huang et al., 2013), suggesting that these inclusions have trapped the mineralizing fluids.

In the next sections, a thermodynamic model is established to examine how fluid pressure affects chemical equilibrium and tungsten solubility in NaCl-H2O-CO2 system. Wood and Samson (2000) proposed a thermodynamic model for understanding the mechanisms precipitating ferberite and scheelite from CO2-free NaCl solutions. Some reactions in their models and CO2-related reactions are considered together in our models. A hydraulic fracturing process is also simulated to constrain the evolution of fluid pressure using finite element based numerical experiments.

3. Methods

3.1. Thermodynamic modeling of a W-bearing NaCl-H2O-CO2 system

Wolframite [(Fe,Mn)WO4] in tungsten deposits in the world is often a complete solid solution between ferberite (Fe2WO4) and hübnerite (MnWO4) (e.g. Harlaux et al., 2018; Pačevski et al., 2007; Sakamoto, 1983; Tindle and Webb, 1989). Both Fe-dominated and Mn-dominated wolframite are identified in the vein-type tungsten deposits of southern China (e.g. Xie et al., 2017; Zhang, 1981; Zhang et al., 2018). Current thermodynamic data are insufficient to calculate the equilibrium constant of MnWO4 dissolution (cf. Robie and Hemingway, 1995); thus, MnWO4 was ignored and only the dissolution of FeWO4 was considered in the models (see reaction 3 in Table 1). Since scheelite (CaWO4) is a minor ore mineral in the vein-type tungsten deposits of southern China, CaWO4 and Ca-related reactions are also absent in the models.

The main tungsten species in NaCl aqueous solutions are $\text{H}_2\text{WO}_4^-$, $\text{HWO}_4^{2-}$, $\text{WO}_4^{3-}$, $\text{NaHWO}_4^-$, and $\text{NaWO}_4^-$. (Redkin and Kostromin, 2010; Wood and Samson, 2000). These species and their reactions with $\text{H}^+$ or $\text{Na}^+$ were incorporated in the models.

Hydrothermal fluids in the crust are generally of low oxidation potential, and dissolved Fe is predominantly in the $+2$ oxidation state (Heinrich and Seward, 1990). It is reported that the two-mica granite in the Dajishan tungsten deposit has an oxygen fugacity of $\log f_{O_2} = -15$ (Jiang et al., 2004). The muscovite granite and the lepidolite granite in the Dahutang tungsten deposit have a lower oxygen fugacity $\log f_{O_2} < -15$ (Han et al., 2016). Under these reduced conditions, the amount of $\text{Fe}^{3+}$ in hydrothermal fluids is negligible compared to that of $\text{Fe}^{2+}$ (cf. Wood and Samson, 2000). Thus, $\text{Fe}^{2+}$ and its reactions with $\text{Cl}^-$ and $\text{OH}^-$ were considered in the models.
Species of oxidized carbon dissolved in water are dominated by carbonate ion (CO$_3^{2-}$), bicarbonate ion (HCO$_3^-$), and dissolved CO$_2$ (aq) (Manning et al., 2013). These oxidized carbon species interact via two stepwise dissociation reactions (see reactions 15 and 16 in Table 1).

21 species and 16 reactions in aqueous solutions were investigated in the models (Table 1). 21 equations were required to be solved for determining the concentration of the 21 species. The equilibrium constants of the 16 reactions in Table 1 were calculated using the R package CHNOSZ developed by Dick (2017). Thus, 16 non-linear equations were established from these 16 reactions. The other five equations were obtained from the charge and mass balance (Table 2). Note that species concentrations were used in the balance equations and species activities were used in the reactions (see Section 3.1.2). The last equation in Table 2 was established based on the assumption that CO$_2$ in NaCl solutions was saturated. The CO$_2$-bearing three-phase fluids inclusions (liquid water, liquid CO$_2$, and gaseous CO$_2$) at room temperature have been identified in several vein-type tungsten deposits of southern China (e.g. Wang et al., 2012; Xi et al., 2008; Xiong et al., 2017; Zhou et al., 2017). The mole fractions of CO$_2$ in CO$_2$-bearing fluid inclusions in the Dajishan and Pangushan tungsten deposits have a range of 0.06–0.76 and 0.22–0.46, respectively (Wang et al., 2012; Xi et al., 2008). Fig. 4 shows the mole fraction of CO$_2$ in CO$_2$-saturated NaCl solutions under the temperature, pressure, and salinity conditions.

Table 1

<table>
<thead>
<tr>
<th>Number</th>
<th>Reaction</th>
<th>LogK (350 °C, 100 MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H$^+$ + HWO$_4^{-}$ = H$_2$WO$_4^+$</td>
<td>3.32</td>
</tr>
<tr>
<td>2</td>
<td>H$^+$ + WO$_4^{2-}$ = HWO$_4^{-}$</td>
<td>6.88</td>
</tr>
<tr>
<td>3</td>
<td>FeWO$_4$(s) = Fe$^{2+}$ + WO$_4^{2-}$</td>
<td>−15.21</td>
</tr>
<tr>
<td>4</td>
<td>Fe$^{2+}$ + Cl$^-$ = FeCl$_2$</td>
<td>2.34</td>
</tr>
<tr>
<td>5</td>
<td>Fe$^{3+}$ + 2Cl$^-$ = FeCl$_2^+$</td>
<td>4.55</td>
</tr>
<tr>
<td>6</td>
<td>H$^+$ + Cl$^-$ = HCl</td>
<td>0.19</td>
</tr>
<tr>
<td>7</td>
<td>Na$^+$ + Cl$^-$ = NaCl</td>
<td>0.79</td>
</tr>
<tr>
<td>8</td>
<td>H$_2$O = H$^+$ + OH$^-$</td>
<td>−10.66</td>
</tr>
<tr>
<td>9</td>
<td>Na$^+$ + H$_2$O = NaOH$^-$ + H$^+$</td>
<td>−10.25</td>
</tr>
<tr>
<td>10</td>
<td>Fe$^{2+}$ + H$_2$O = FeOH$^+$ + H$^+$</td>
<td>−4.49</td>
</tr>
<tr>
<td>11</td>
<td>Fe$^{3+}$ + H$_2$O = Fe$^3$O$^2+$ + 2H$^+$</td>
<td>−10.23</td>
</tr>
<tr>
<td>12</td>
<td>Fe$^{2+}$ + 2H$_2$O = HFeO$_2^{-}$ + 3H$^+$</td>
<td>−17.13</td>
</tr>
<tr>
<td>13</td>
<td>Na$^+$ + HWO$_4^{-}$ = NaHWO$_4^+$</td>
<td>0.15</td>
</tr>
<tr>
<td>14</td>
<td>Na$^+$ + WO$_4^{2-}$ = NaWO$_4$</td>
<td>2.93</td>
</tr>
<tr>
<td>15</td>
<td>CO$_2$ (aq) + H$_2$O = HCO$_3^-$ + H$^+$</td>
<td>−8.30</td>
</tr>
<tr>
<td>16</td>
<td>HCO$_3^-$ = CO$_2^{2-}$ + H$^+$</td>
<td>−11.06</td>
</tr>
</tbody>
</table>

The LogK of the reaction 3 was calculated using the function and the thermodynamic data in Wood and Samson (2000).
of mineralizing fluids in the vein-type tungsten deposits of southern China. The mole fractions of CO2 are generally lower than 0.4. Thus, the assumption of CO2 saturation in hydrothermal fluids is plausible. CO2 solubility in NaCl aqueous solutions was reproduced from the CO2 solubility model proposed by Mao et al. (2013). These 21 nonlinear equations were solved using the R package rootSolve developed by Soetaert (2016).

3.1.2. Activity coefficients of charged and neutral species

The activity of a species in electrolyte solutions is its effective concentration and equals its concentration multiplied by its activity coefficient. The activity coefficients of electrically charged species in NaCl aqueous solutions are often calculated by an extended Deby-Hückel equation (B-dot equation) proposed by Helgeson (1969) (e.g. Gibert et al., 1992; Mikucki, 1998; Wood and Samson, 2000):

\[
\log \gamma_i = \frac{-A_{z_i}^2 \sqrt{I}}{1 + aB \sqrt{I}} + b \tag{1}
\]

in which \( \gamma_i \) is the activity coefficient of the \( i \)th ion, \( z_i \) is the charge number of the ion, \( I \) is the ionic strength, \( A \) and \( B \) are two parameters related to the dielectric constant and density of water, and \( a \) and \( b \) are temperature-dependent constants. The ionic strength of a solution is a function of the concentrations of all the ions in that solution:

\[
I = \frac{1}{2} \sum_{i=1}^{6} c_i z_i^2 \tag{2}
\]

in which \( I \) is the molar ionic strength in mol/kg and \( c_i \) is the molar concentration of the \( i \)th ion. The only species-specific parameter required in equation (1) is the electrical charge. This treatment may be incorrect, but its errors are negligible compared to those caused by other sources (Zhu et al., 2017). Activity coefficients of charged species were calculated by the R package CHNOSZ (Dick, 2017). The ionic strength \( I \) had an initial value of half of the molar concentration of NaCl in solutions and was updated in an iterative method until the absolute error was less than 0.01.

Neutral species are considered to mix ideally in the solutions in Deby-Hückel type models (cf. Walther, 1997); therefore, activity coefficients of neutral aqueous species are often assumed to be unity (e.g. Gibert et al., 1992; Helgeson et al., 1981; Wood and Samson, 2000). However, when neutral species are high-concentration, the non-ideal interactions between neutral species and charged species may be considerable (cf. Walther, 1997). CO2 is highly concentrated if CO2 is saturated in NaCl aqueous solutions (see Fig. 4). The empirical equation fitted by Drummond (1981) was used to reproduce the activity coefficients of CO2 (see equation A-10 in Spycher and Pruess, 2005). Fig. 5 shows that the activity coefficient of CO2 tends to unity when the salinity approaches zero. The activity coefficients of the other neutral species in our models were assumed to unity given that reliable data for their activity coefficients are not available.

![Fig. 4. The solubility (a) and mole fraction (b) of CO2 in NaCl aqueous solutions reproduced from the model proposed by (Mao et al., 2013). The solubility of CO2 decreases with increasing salinity of the solutions. This is called the salting-out effect (Dubocq et al., 2013; Nighswander et al., 1989).](image)

![Fig. 5. Activity coefficients of CO2 in NaCl aqueous solutions reproduced from the model developed by Drummond (1981).](image)
3.2. Numerical modeling of hydraulic fracturing

Hydraulic fracturing is a fluid-to-solid coupling where a change in fluid pressure or fluid mass alters the volume of a porous material and produces strains (Wang, 2000). In the Navier-Coulomb criteria, rock failure occurs by hydraulic fracturing when the differential stress $\sigma_{d} = \sigma_{l} - \sigma_{v}$ is less than four times tensile strength $T_{s}$ and the effective minimum principle stress $\sigma_{1} = \sigma_{l} - P$ reaches $T_{s}$ (Cosgrove, 1995; Cox, 2007; Plimer, 1987; Tsuchiya et al., 2016). The fluid pressure fluctuations after hydraulic fracturing may be recorded by fluid inclusions in veins (e.g. Rusk et al., 2004). Fracturing driven by high pressure fluids may also trigger earthquakes (e.g. Miller et al., 2004).

The orientation of the quartz veins in the tungsten deposits of southern China indicates that their tension direction is approximately north–south striking. Our previous study indicates that the maximum principle stress $\sigma_{1}$ was vertical and the intermediate principle stress $\sigma_{2}$ was east–west striking (Liu et al., 2014). Structural analysis of the vein arrays suggests that the initiation and propagation of the fractures were triggered by high-pressure fluids (e.g. He and Xi, 1988; Wang et al., 2008; Wei et al., 2015; Yu, 2004). In situ fragmentation textures in the veins (Fig. 3a) suggests that the brecciation is fluid-assisted (cf. Jebrak, 1997). Fluid inclusions from several tungsten deposits also record high-pressure fluids (e.g. Huang et al., 2006; Wang et al., 2007; Wang et al. 2013a Xi et al., 2008). Fluid-driven fracturing is modelled in this part to examine the change of fluid pressure during a hydraulic fracturing process and to be coupled to the thermodynamic model in Section 3.1.

The maximum pressures recorded in fluid inclusions were assumed to reach lithostatic pressure levels (25 MPa/km). Thus, the tungsten deposits in southern China may form at a depth of 3–6 km. A two-dimension hydro-mechanical model at a depth of 4 km was built. The model has a size of 100 m x 100 m (Fig. 6). X axis represents the orientation normal to veins and Z axis is vertical. The coupling of rock deformation and fluid flow in this study was governed by poro-elastic constitutive equations and the diffusion equation of fluid pressure (see Appendix). These simultaneous partial differential equations were solved by an in-house finite element-based supercomputer simulator named PANDAS (Li and Xing, 2015; Liu et al., 2017a; Xing and Makinouchi, 2002). The yield criteria used in the models are the Maximum Tensile Stress Criterion and the Mohr-Coulomb Criterion (see Appendix).

Isothermal fluids were assumed to be injected into the bottom of the models. The aim of this treatment is to save computational power and focus on the influence of fluid pressure on tungsten solubility. High-pressure fluids at 350 °C were used in the models and this temperature is within the homogenization temperatures of the tungsten deposits. The constitutive equations used in our models are independent on temperature (see Appendix).

The fluid physical properties required in the models include fluid density, compressibility, and viscosity. Mao et al. (2015) proposed a predictive model that reproduces the single-phase density of NaCl-H2O-CO2 fluid mixtures of all compositions from 273 to 1273 K and from 1 to 5000 bars within experimental uncertainty in most cases. Their model was used to reproduce the fluid density and fluid compressibility in our models. Current models for fluid viscosity of CO2-bearing NaCl aqueous solutions are only valid at T ≤ 60 °C, P ≤ 300 bars (e.g. Bando et al., 2004; Fleury and Deschamps, 2009; Islam and Carlson, 2012; Kumagai and Yokoyama, 1999). Klyukin et al. (2017) developed an empirical model for the viscosity of H2O-NaCl fluids at T ≤ 1000 °C, P ≤ 5000 bars, and salinity ≤ 100 wt% NaCl. Their model was used to reproduce the fluid viscosity in our models. Note that the fluid physical properties above were fixed to a constant level but were varied in different numerical experiments to examine their influences on hydraulic fracturing.

The fluids released from a CO2-bearing magma are early low-salinity CO2-rich fluids followed by more saline CO2-free fluids (Baker, 2002; Holloway, 1976). The CO2-bearing fluid inclusions in wolframite in the tungsten deposits have a salinity of 0.4–3.6 wt% NaCl equivalent (Chen et al., 2018; Li et al., 2018), and 1–6 wt% NaCl equivalent in quartz (Wang et al., 2012; Xi et al., 2008). The CO2-free fluid inclusions have a salinity up to 10 wt% NaCl equivalent (e.g. Ni et al., 2015). Thus, a salinity of 10 wt% NaCl equivalent was used in the numerical experiments. This salinity is an upper limit of the fluid salinities above.

The quartz veins in the tungsten deposits of southern China are hosted by low-porosity sandstone and slate, granite (Li, 1993; Zhang et al., 2017; Zhao et al., 2017b). The rock in the models was assumed to be unfractured and had an initial permeability of 10^(-16) m^2. Equation (A-8) in Appendix shows that the rock permeability decreases with the effective normal stress before fracturing; therefore, the rock permeability before fracturing was lower than the minimum permeability (10^(-16) m^2) required for advective heat transport (cf. Manning and Ingebritsen, 1999). This treatment prevents heat transfer from hot fluids to the rock and is consistent with the assumption of isothermal fluids. The rock in our models has a Young’s modulus of 60 GPa, a Poisson’s ratio of 0.2, and a tensile strength of 5 MPa according to the experimental data in Pariseau (2006), Gereck (2007), and Lockner (1995), respectively. Table 3 shows the rock mechanical parameters and the fluid properties used in the numerical experiments.

The finite element based numerical experiments were run in two stages. At the first stage, a vertical compressive stress of 100 MPa ($\sigma_{v}$) was loaded at the top, a horizontal compressive stress of 50 MPa ($\sigma_{h}$) was loaded on the left and right sides of the model, and the bottom was fixed vertically (Fig. 1). This corresponds to a horizontal-over-vertical stress ratio of $\frac{\sigma_{h}}{\sigma_{v}} = 0.5$. These boundary conditions formed an initial extensional stress field at a depth of 4 km if a lithostatic gradient of 25 MPa/km was assumed. At the second stage, the loaded stresses at the first stage were maintained and fluids with a fixed pressure of 200 MPa were released at the bottom. A small amount of fluids was assumed to exist in rocks before release of high-pressure magmatic fluids; thus the model was initially unsaturated and had an initial fluid pressure of 10 MPa. Fig. 7 shows the finite element meshes consisting of 39,200 elements and 59,643 nodes. A reference point shown in Fig. 7 was chosen to show the change of the effective minimum principle stress and the fluid pressure during a hydraulic fracturing process.

Six numerical experiments were conducted in this study and their parameters are shown in Table 3. Fluid properties at 350 °C, 200 MPa,

![Fig. 6. A 2D geometric model of hydraulic fracturing at a depth of 4 km.](image)
show the influences of fluid compressibility on hydraulic fracturing.

Fig. 8 shows that the effective minimum principle stress at the reference node decreased as fluid pressure increased in E1. The effective mean normal stress was initially compressive and increased with fluid pressure. The permeability at the reference node increased with the effective mean normal stress and was lower than $10^{-16}$ m$^2$ before fracturing. The effective minimum principle stress reached the tensile strength of 5 MPa after 2.7 s when fluid pressure reached 167.95 MPa. The peak fluid pressure reached at the time when the reference node yields is also called the breakdown pressure (Bunger et al., 2010). After fracturing, the permeability increased to $2 \times 10^{-13}$ m$^2$ and the fluid pressure fluctuated significantly and decreased to 109.60 MPa after 2.8 s. In PANDAS, the stresses become zero once they reach the shear or tensile strength. In contrast, the tensile strength at the reference point in E2 was satisfied after 0.84 s, earlier than in E1. After fracturing, fluid pressure fluctuated from 156.33 MPa to 121.42 MPa.

4.2. Influences of fluid viscosity on hydraulic fracturing

The empirical model for fluid viscosity used in this study is independent on CO$_2$ concentration. Previous viscosity measurements of aqueous NaCl solutions with dissolved CO$_2$ suggests that fluid viscosity is positively correlated to CO$_2$ concentration (e.g. Bando et al., 2004). Thus, fluid viscosity in the third numerical experiment (E3) was increased by 10% based on that in E1 and the other parameters remained unchanged.

The effective minimum principle stress at the reference node in E3 was higher than that in E1 at any time before fracturing (Fig. 9). The tensile strength in E3 was satisfied after 3.0 s, later than that in E1. The fluid pressure at the reference node in E3 also shows significant fluctuations after fracturing. The breakdown pressure in E3 is 167.99 MPa, slightly higher than in E1. Fluid pressure in E3 decreased to 87.91 MPa after 3.1 s.

4.3. Influences of the initial fluid pressure on hydraulic fracturing

The initial fluid pressure is another uncertain variable when high-pressure fluids are released. The initial fluid pressure in the fourth numerical experiment (E4) was increased to 15 MPa based on the first one and the other parameters kept unchanged.

The effective minimum principle stress at the reference node in E4 was always larger than that in E1 before fracturing and the tensile

![Fig. 7. The finite element meshes used in the numerical model of hydraulic fracturing. A reference node close to the fixed-pressure fluid source was chosen to illustrate the evolution of the stresses and the fluid pressure during a hydraulic fracturing process.](image-url)
The horizontal stress is a critical parameter affecting the initial stress field (e.g., Rutqvist et al., 2013). This parameter was increased to 55 MPa in the fifth numerical experiment (E5) to study how a higher horizontal stress affects hydraulic fracturing. The reference node in E5 had a higher initial effective minimum principle stress than that in E1 (Fig. 11). The decrease of the effective minimum principle stress in E5 was slower than that in E1 and the tensile strength was satisfied after 7.2 s. The fluid pressure fluctuated from 175.15 MPa to 122.32 MPa.

4.5. Influences of the fixed fluid pressure on hydraulic fracturing

The last parameter analyzed in this study is the fixed fluid pressure at the bottom of the model. The fixed fluid pressure in the sixth numerical experiment (E6) was decreased to 190 MPa and the other parameters remained unchanged compared to those in E1. The tensile strength of the reference node in E6 was satisfied after 4.4 s, later than that in E1 (Fig. 12). The breakdown pressure was 164.17 MPa and then the fluid pressure fluctuated to 92.01 MPa.

4.6. Chemical equilibrium of a tungsten-bearing NaCl-H₂O-CO₂ system

Three concentrations of CO₂ in NaCl solutions at 350 °C were used in solving the nonlinear equations in Section 3.1. The first concentration is CO₂ solubility, the second one is 50% of CO₂ solubility, and the third is 10% of CO₂ solubility (Fig. 13). pH varies from 3.3 to 4.7 and decreases with increasing fluid pressure and the concentration of CO₂ in
solutions. Tungsten solubility increases with fluid pressure and the concentration of CO2 in solutions and changes from 23 to 50 ppm.

The dominant iron species are FeCl20, FeCl+, and Fe2+, the concentrations of which are 2–10 orders of magnitude higher than those of FeOH+, FeO0, and HFeO2− (Fig. 14). The concentrations of FeCl20, FeCl+, Fe2+, and FeOH+ increase with fluid pressure, while those of FeO0, and HFeO2− decrease with fluid pressure.

Tungsten species are dominated by HWO4− and NaWO4− and their concentrations are 1–2 orders of magnitude higher than those of NaHWO40, H2WO40, and WO42− (Fig. 14). The concentration of HWO4− is positively correlated to fluid pressure, while the concentration of NaWO4− has a negative correlation to fluid pressure. The source data can be accessed from the supplementary table in the web version.

4.7 Chemical responses to hydraulic fracturing driven by CO2-saturated fluids

Table 4 shows the change of pH and tungsten solubility in the first, third, fourth, and the fifth numerical experiments. The fluid pressure fluctuated over ten times in each numerical experiment. pH in these four numerical experiments had a range of 3.42–3.91 and increased by 0.32 on average. The increase in pH resulted in an accumulated decrease of 12.61–16.25 ppm in tungsten solubility, which accounted for 31.05% of the maximum tungsten solubility on average.

5. Discussion

5.1 Validity of numerical experiments of hydraulic fracturing

Magmatic-hydrothermal fluids are highly compressible and have a significantly low viscosity compared to H2O-bearing silicate melts (Audétat and Keppler, 2004). Previous studies have identified that fluid compressibility has an influence on seismic activities (e.g. Geli et al., 2014; Geli et al., 2016). Findings from the field of oil and gas industry suggest that an increase in fluid compressibility and fluid viscosity slows the change of the effective stresses and prolongs the time of...
fracturing (e.g. Chen et al., 2015; Ishida et al., 2004). This is consistent with our numerical experiments (E1, E2, and E3).

The classic equations for analyzing the breakdown pressure suggest that this parameter is positively correlated to the initial pore pressure and the minimum horizontal stress (cf. Burger et al., 2010; Kumari et al., 2018). This is in accord with the results of the fourth and fifth numerical experiments. The sixth numerical experiment (E6) had a lower fixed fluid pressure than the first one (E1). A low fixed fluid pressure caused a low pressurization rate (Fig. 12c). The breakdown pressure in E6 was lower than that in E1. Findings from laboratory and numerical experiments of hydraulic fracturing suggest that the breakdown pressure increases with the pressurization rate (e.g. Detournay and Cheng, 1992; Zhuang et al., 2018; Zoback et al., 1977). This is consistent with the difference of the breakdown pressure between E6 and E1.

Hydraulic fracturing in the numerical experiments was triggered at a time scale of a few seconds. The driving fluid pressure used in the numerical experiments is two times the lithostatic pressure at that depth. The fluid overpressure formed in crust often falls between the hydrostatic level and the lithostatic level at a given depth (Peacock et al., 2017). Comparisons between the first and sixth numerical experiments indicate that a decrease in the driving fluid pressure prolongs the time of fracturing. Therefore, the timescales of hydraulic fracturing during formation of the tungsten deposits in southern China may be longer than those in the numerical experiments. Certainly, hydraulic fracturing is a transient mechanical process and its timescales are shorter than those of hydrothermal flow and chemical reactions in hydrothermal systems (Cox, 2005; Cox, 2016; Jébrak, 1997; Zhao et al., 2012). A typical example of this transient process is fluid-driven earthquakes (e.g. Miller et al., 2004).

5.2. Comparisons with other thermodynamic models and experimental results

pH in the numerical experiments of hydraulic fracturing is 3.42–3.91, which is lower than that (pH = 4–6) of the mineralizing fluids forming tungsten deposits (Wood and Samson, 2000). This difference may be caused by the assumption that CO2 dissolved in NaCl solutions is saturated in the thermodynamic model. Note that the pH data in Wood and Samson (2000) come from the tungsten deposits in the world except China and no pH of the tungsten deposits of southern China is reported until now.
Polya (1990) found from a thermodynamic model that tungsten solubility decreases with decreasing fluid pressure. The tungsten-bearing species in his model are $\text{HWO}_4^{-}$ and $\text{WO}_4^{2-}$. More tungsten-bearing species were considered by Wood and Samson (2000). The dominant tungsten species in their models is $\text{HWO}_4^{-}$, which is consistent with our models. Tungsten solubility in their models reaches a few hundred ppm, which is higher than that in our models. This difference comes from that scheelite and Ca-related reactions are absent in our models. Their thermodynamic models suggest that tungsten solubility is not a monotonic function of fluid pressure. In contrast, tungsten solubility is positively correlated to fluid pressure in our models. The comparisons above indicate that the results strongly depend on the tungsten species and the existence of CO$_2$ in hydrothermal fluids.

Fig. 14. The change of iron species and tungsten species against fluid pressure in NaCl solutions with different CO$_2$ concentrations. The dominant iron species and tungsten species are $\text{FeCl}_2^0$ and $\text{HWO}_4^{-}$, respectively.

<table>
<thead>
<tr>
<th>Breakdown pressure (MPa)</th>
<th>Average</th>
<th>E1</th>
<th>E3</th>
<th>E4</th>
<th>E5</th>
</tr>
</thead>
<tbody>
<tr>
<td>pH range</td>
<td>[3.45, 3.76]</td>
<td>[3.45, 3.91]</td>
<td>[3.43, 3.69]</td>
<td>[3.42, 3.68]</td>
<td></td>
</tr>
<tr>
<td>pH increase in log unit</td>
<td>0.31</td>
<td>0.46</td>
<td>0.26</td>
<td>0.26</td>
<td>0.32</td>
</tr>
<tr>
<td>Fluctuation times</td>
<td>11</td>
<td>14</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Maximum tungsten solubility (ppm)</td>
<td>43.41</td>
<td>43.42</td>
<td>44.01</td>
<td>44.73</td>
<td>44.73</td>
</tr>
<tr>
<td>Accumulated tungsten decrease (ppm)</td>
<td>12.61</td>
<td>16.25</td>
<td>12.79</td>
<td>12.82</td>
<td>13.62</td>
</tr>
</tbody>
</table>

Table 4
The change of pH and tungsten solubility in the numerical experiments of hydraulic fracturing.
Liu et al. (2017b) conducted crystallization experiments of wolframite in a hydrothermal diamond-anvil cell using [Fe,Mn]WO$_4$-Li$_2$CO$_3$-H$_2$O as starting materials. They found that solubility of wolframite increased with CO$_2$ component in solutions. Therefore, our thermodynamic results are consistent with their findings although Mn-related reactions are absent in our models.

5.3. Hydraulic fracturing and wolframite precipitation

The WO$_3$ grade in the vein-type tungsten deposits of southern China generally reach 0.15–1.5% (Mao et al., 2013), which are 10$^4$–10$^5$ times the Clarke value of 0.6 part per million for the crust in eastern China (Chi et al., 2012). However, it is poorly understood how the ores are precipitated from hydrothermal fluids (e.g. Liu et al., 2015; Polya, 1988). The similar question is also pending in other types of hydrothermal deposits (e.g. Heinrich et al., 2005; Henley and Berger, 2000; Zhao et al., 2017a).

The mineralizing fluids of the tungsten deposits are interpreted to have a magmatic source (e.g. Chen et al., 2018; Li et al., 2018). In this case, simple cooling is inefficient to precipitate minerals because rocks have a low thermal conductivity (Barton and Toumlin, 1961; Heinrich, 2005; Heinrich and Candela, 2014). Over ten fluctuations of fluid pressure were identified in our numerical experiments. The first fluctuation was caused by a significant increase in the permeability of the reference node. Repeated recovery of fluid pressure resulted in fracturing of the surrounding nodes and the following fluctuations of fluid pressure. The fluid pressure drops in our numerical experiments were smaller than that caused by a seismic slip (Weatherley and Henley, 2013). However, repeated fluctuations of fluid pressure during a hydraulic fracturing process still decreased the solubility of CO$_2$ and caused CO$_2$ escaping from hydrothermal fluids. CO$_2$ escaping increased pH and broke the chemical equilibrium where the dominant iron-bearing and tungsten-bearing species decreased. The accumulated tungsten precipitated after hydraulic fracturing reached 12.61–16.25 ppm, which accounted for approximately 30% of the maximum tungsten solubility in solutions. Therefore, hydraulic fracturing is an efficient process for precipitating wolframite in the tungsten deposits.

5.4. Limitations and further improvements

The numerical models in this study are established on many assumptions and provide an explanation for the mechanisms of precipitating wolframite from CO$_2$-saturated hydrothermal fluids. Further modifications are suggested for improving the models and better explaining how wolframite precipitates from hydrothermal fluids.

First, wolframite in the tungsten deposits in southern China is a solid solution between the two end-members, ferberite (FeWO$_4$) and hüebnerite (MnWO$_4$) (e.g. Wang and Ji, 1989; Xie et al., 2017). Thermodynamic data related to MnWO$_4$ and Mn$^{2+}$ are required for further improving the thermodynamic model of tungsten solubility in hydrothermal fluids.

Second, CO$_2$-bearing fluid inclusions are not identified in all the tungsten deposits (cf. Ni et al., 2015). However, it remains poorly understood how wolframite precipitates from CO$_2$-free hydrothermal solutions that are not mixed with meteoric fluids. This issue also met in other types of hydrothermal deposits (e.g. Heinrich et al., 2005). An attempt to solve this issue is to consider the concept of fluid mixing proposed by Lester et al. (2012) and examine whether and how chaotic advection of reactive hydrothermal fluids causes mineral deposition.

Third, fluid temperature was assumed to be independent on fluid pressure change in our models. Whether a decrease in fluid pressure causes a decrease in fluid temperature should be examined because fluid temperature is a key variable controlling solubility of tungsten and other metals in hydrothermal fluids (Yardley, 2005). This question is related to adiabatic decompression (Heinrich et al., 2005), which is quantified by the Joule-Thompson coefficient (Stauffer et al., 2014). However, the Joule-Thompson coefficients of NaCl aqueous solutions over 300°C and 100 MPa are unknown (Wood and Spera, 1984). Determination of this coefficient at higher temperatures and pressures may help us find the mechanisms precipitating wolframite and other ore minerals in magmatic-hydrothermal deposits.

6. Conclusions

Finite element based numerical experiments of hydraulic fracturing coupled with a multi-component thermodynamic model were conducted to examine whether the change of fluid pressure during a hydraulic fracturing process could cause wolframite precipitation. CO$_2$-bearing aqueous NaCl solutions with a salinity of 10 wt% at 350°C were used in the models. Hydraulic fracturing in the numerical experiments is driven by fluids at a fixed fluid pressure. The changes of fluid pressure in the numerical experiments are linked to the thermodynamic model. The numerical experiments provide the following implications for the mechanisms precipitating wolframite in the tungsten deposits of southern China:

1) Hydraulic fracturing is influenced by fluid compressibility, fluid viscosity, the initial fluid pressure, the horizontal stress, and the fixed fluid pressure. An increase in CO$_2$ concentration increases the fluid compressibility and prolongs the time of hydraulic fracturing.

2) CO$_2$ solubility in NaCl aqueous solutions is positively correlated to fluid pressure. A decrease in fluid pressure increases pH and reduces the concentrations of the dominant iron species and the dominant tungsten species. Therefore, tungsten solubility decreases with decreasing fluid pressure. The pH derived from the thermodynamic model is between 3.3 and 4.7 depending on fluid pressure and CO$_2$ concentrations. Tungsten solubility in fluids at 350°C reaches tens of ppm.

3) Over ten fluctuations of fluid pressure were identified during a hydraulic fracturing process. These fluid pressure fluctuations could cause a decrease in tungsten solubility by 12.61–16.25 ppm, which accounts for 31.05% of the maximum tungsten solubility. Thus, hydraulic fracturing is an efficient process for precipitating wolframite from CO$_2$-bearing hydrothermal fluids.

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A. Appendix

Basic equations

Elastic-plastic finite deformation theory is used in PANDAS (Xing and Makinouchi, 2002). The rate of deformation tensor is supposed to be the sum of an elastic part \( D^e \) and a plastic part \( D^p \):

\[
D^f = D^e + D^p
\]

which are prescribed by the Hookes law and the associated flow rule, respectively. The elastic strain increment \( D^e \) is:

\[
D^e = \frac{\partial \sigma^e}{\partial \varepsilon^e}
\]

in which \( \sigma^e \) is the Jaumann derivative of \( \sigma^e \).

The associated flow rule is as follows:

\[
D^p = \dot{\lambda} \frac{\partial f}{\partial \sigma^e}
\]

in which \( \dot{\lambda} \) is the plastic multiplier, \( \sigma^e \) is the Cauchy stress, and \( f \) is a plastic potential. Both the Maximum Tensile Stress Criterion and the Mohr-Coulomb Criterion are applied here.

The effective stress \( \sigma^e \) is used when fluids are filled in the pores:

\[
\sigma^e = \sigma - \alpha p \delta j
\]

in which \( P \) is the pore fluid pressure, \( \alpha \) is the coefficient of the pore pressure, and \( \delta j \) is the Kronecker delta.

Following the mass conversation theory, the pressure diffusion equation is:

\[
k \nabla^2 p = \frac{1}{Q} \frac{\partial p}{\partial t} - a \frac{\partial \varepsilon^e}{\partial t}
\]

\[
\frac{1}{Q} = \mu S
\]

\[
S = \frac{1}{K_i} + \frac{\phi}{K_f}
\]

in which \( k \) is the permeability scalar, \( Q \) is the Biot constant, \( \varepsilon^e = \varepsilon^i + \varepsilon^d + \varepsilon^c \) is the volume strain, \( \mu \) is the fluid viscosity, \( \phi \) is the porosity, \( K_i \) is the bulk modulus of the fluid and equals the reciprocal of fluid compressibility, and \( K_f \) is the bulk modulus of the solid matrix.

The permeability scalar follows the exponential law of effective stress (David et al., 1994):

\[
k = k_0 \exp \left( -\omega \left( \frac{\sigma^e}{3} - \alpha P \right) \right)
\]

\[
\frac{k}{k_0} = \left( \frac{\sigma^e}{\phi_0} \right)^{\frac{1}{\xi}}
\]

in which \( k_0 \) is the initial permeability scalar, \( \phi_0 \) is the initial porosity, \( \omega \) is the effective pressure sensitivity coefficient, \( \frac{\partial \sigma^e}{3} - \alpha P \) is the effective mean normal stress, and \( \xi \) is a damage factor. \( \xi = 1 \) when the rock is in elastic state. Low-porosity crystalline rocks have an effective pressure sensitivity coefficient of \( 10^{-2} \) to \( 10^{-2} \) MPa\(^{-1} \) (David et al., 1994). \( \omega = 5 \times 10^{-3} \) was used in our models. Findings from laboratory and field experiments of hydraulic fracturing suggest that the rock permeability is increased by at least \( 10^3 \) times after fracturing (e.g. Evans et al., 2012; Ladner and Häring, 2009; Watanabe et al., 2017).

In our models, \( \xi = 2000 \) is used when one of the two criterions above is met.

Appendix B. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.oregeorev.2018.08.027.

References


